World Oil Depletion Models: Price Effects Compared with Strategic or Technological Interventions

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Running title: World Oil Depletion Models

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Preprint submitted to Elsevier Science *2 February 2006*

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Abstract

World oil depletion, including natural gas liquids, was modelled in the past by many authors. Recently, Guseo and Dalla Valle have introduced and Guidolin has applied a new approach following perturbed life–cycle diffusion models. Here we examine joint effects of economic and strategic or technological interventions using a Generalized Bass Model (GBM). Statistical analysis takes into account three different hierarchical levels: natural diffusion, long memory interventions and stochastic components. The main results confirm the statistical significance of historical 1970's shocks and highlight a strong long memory effect due to an increase in oil production after World War II. The estimated peak–date, 2007, and the 90% depletion time, 2019, are determined under an equilibrium intervention hypothesis.

Key words: Oil peak, Depletion times, Oil price elasticity, Diffusion process, Generalized Bass model, Nonlinear models, ARMAX

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1 Introduction

The powerful worldwide economic growth after World War II has been sustained by *surplus capacity* in hydrocarbon fuels, in particular by crude oil intensive production. If we look at Figure 1 we can observe that world daily crude oil extraction increased with a weak proportional shape from the beginning of the 20th century until the end of World War II. We note that, after this long period, there was an exponential deviation of daily oil production sustained for at least two decades until the well–known international crude oil shocks of the 1970's. Growth and development of efficient new technologies within transport, heating, cooling, chemical products for industry and agriculture are essentially based upon crude oil transformations.

Emerging economies such as China, India and other countries are now increasing the new global demand for more energy (see, for example, Figure 8). China is strongly engaged in coal extraction in order to sustain the internal electric increasing demand of a day by day expanding economy.

The expansion of world economy interacts with the evolution of population.

The demographic effects of the over rapid economic growth are well described by Cohen [3]. The trend toward urbanization is clear. The rural population of the rich countries peaked around 1950 and has slowly declined since then. World average length of life rose from about 30 years at the beginning of the 20th century to 65 years at the beginning of the 21st century. Current global population growth rates are far higher than any experienced before World War II. The world population of 6.3 billion will peak, around 2050, at 8.9 billion (medium variant scenario of United Nations Population Division, World Population Prospects: the 2002 Revision, Highlights). At the same time, thirty of the more developed countries are expected to have a lower population in 2050 than today (Japan −24%, Italy −22%, FSU −29%). Migrations, food, housing, education, health, employment and public order pose formidable challenges to economies, and to social and political governance, that should conceive efficient technological innovations to overcome these crucial problems.

This issue was previously examined by Marchetti (see [4], [5], [6]). He highlighted that *niche* of mankind is expanding under the new advances of technology even if diffusion of population is not homogeneous, especially in Europe. Nevertheless, some regularities in the evolution of inventions and innovations in waves – supported by new energy resources – are strongly promising and contrary to Malthusian approaches. Information, knowledge and science, as analyzed by Marchetti [6], are the new paradigm of *Logos*.

During World War II, USA dominance in production and refining of petroleum, since the end of World War I, provided a decisive military advantage (see for instance Molitor [7]). USA supplied 90% of the Allied Forces. Infrastructure development accelerated demand. New Jersey high speed and limited access roadways, Holiday Inns highway motels, fast-food and drive–in are worldwide known symbols of the 1950's.

FIGURE 1 ABOUT HERE

Technology has generally led to a greater use of hydrocarbon fuels for most human activities making civilization vulnerable to decreases in supply as highlighted by Hall et al. [8]. The strong economic growth after World War II seems to be the foundational basis of a structural and irreversible modification of global economy and correlated energy consumptions.

Nowadays, the most developed economies are facing the beginning of capacity limitations so that the assumption of an increasing oil production in the next decade does not seem realistic (see, for istance, Morse and Jaffe [9]). Emerging technologies are not yet commercially or technologically viable to ease completely the shortage even if some hopeful signs are coming out, see for instance Marchetti [5] and Guidolin [2].

In this paper we do not investigate correlated aspects on the dynamics of energy systems and the evolution of energy services nor the dynamics of energy sources including, for instance, the abiogenic hypothesis. Part of these issues are treated in a very huge literature. Alternatives to crude oil are well–known. Coal resources are vast and more equally distributed, heavy crudes, tar sands, deepwater oils are potential candidates with high–oil prices. Gas is offering a much cleaner alternative and probably it will be an efficient substitute (see, for instance [1]). Nuclear power plants and renewable technologies may be possible but not exclusive solutions.

The main question here is not to suggest which one or which cluster of admissible alternatives may be scientifically, technologically and economically sustainable. This stands outside the purposes of this paper.

Our aim is to understand better which is the temporal shape and the extension of the oil production process. This is, in our opinion, a common basis for evaluating multivariate aspects related to an energetic migration. Our attention is intentionally focused on oil production simply because it is difficult to find an aspect of our modern life that is not structurally dependent on oil based energy. Even if it is certainly true that oil is not irreplaceable and hard limitations in its production could offer the opportunity to develop other and, perhaps, better energy sources, this change would inevitably affect our lives in many ways. It would be surely a discontinuity. And it is true that the discontinuities, rather than the intervening calms, shape our destiny. In any event, we can say that those who fail to react to discontinuities will suffer; those who react survive; but those who anticipate prosper (see [14]).

The pioneer of historical evaluations on crude oil depletion is Hubbert [11]. In 1956 he correctly estimated that USA oil production would peak (maximum instantaneous production) around 1970. Later, in [12], he recognized the explicit role of the logistic Verhulst equation (see [13]) in his previous model. More recent geological studies confirm that a world oil peak is imminent. In particular, Colin J. Campbell, president of ASPO (*Association for the Study of Peak Oil and Gas*), published in 1998 with J.H. Laherrère a famous paper on this topic (see [14]) where he emphasized the limited reliability of "technical" evaluation of reserves because of their overestimation for financial reasons and noted that the declared oil reserves in the Middle East raised during 1986–90, without any new discovery. From 1998 until now the debate on oil depletion has become broader with several contributions that propose analyses and predictions of the date of oil's peak. Many of these models refer to Hubbert's approach, trying to extend and update it.

For example, in [15], [16] there are some attempts by Reynolds to include prices and costs in the Hubbert's model. The most interesting is the second work in which an extension – represented by a linear regressive approach – is based upon a discretization of the logistic equation. The proposed approach does not give an easy way to evaluate temporal forecasts. Nevertheless, there are some interesting comments about the inelasticity of oil demand with reference to the prices.

In several papers Laherrère uses the logistic equation (or other equivalent transformations) in order to model discoveries, production and co–evolution of population growth. In [17] and [18] there are considerable efforts in applying a multi–Hubbert modelling. However, the proposed fitting to the more recent part of crude oil production series is not correct because the method erases the memory of old extraction processes with an improper magnification of the residual reserves. Statistical analysis of the properties of the applied methods is omitted.

In [19] Bardi provides a class of new models for the representation of a decline in oil production more abrupt than the growth. Description and analysis are based on a simulated procedure which is not so easy to evaluate with reference, for example, to the persistence of strategies or to an improvement of technological factors. Statistical application is again not the main focus.

One of the most debated aspect of oil depletion models is the notion of *Ultimate Recoverable Resource* (URR) which is a subset of the corresponding *Physical Resource.*

Without entering into details, it should be useful to remind some basic concepts on the origin of this Physical Resource. The formation of oil required a very long and complex process of transformation of organic material (algal material mainly), characterized by specific climatic and physical conditions, whose replication is not simple. As it is well–known, the conditions that led to the deposition and preservation of prolific oil source rocks were extremely rare in the geological record both in time and place. Moreover, these transformations required million of years to be realized, i.e. a time inconceivable for the human kind. Only in the last twenty years advances in geochemistry have made possible to identify where and when oil was generated, emphasizing its very finite nature. For this reason, we really have to consider oil as a not renewable resource.

Ultimate Recoverable Resource is the total amount of a finite resource which may be obtained *at the end* of extraction or production process.

However, geologists have proposed tentative physical *estimates* of the URR *during* the life–cycle of resource extraction. These procedures do not exhibit a general scientific agreement. Their oil estimated URR is a weighted sum of different components. The weights take into account the relative uncertainty of the terms that are associated.

Usually, the geological oil estimated URR includes:

a) *production to date*. This is a historical empirical component with limited uncertainties if we exclude heterogeneity of "oils". Its weight is normally set to one (100%);

b) *proven reserves.* These are oils that we reasonably consider to be able to extract in the future from known *physical resources*, with known techniques and in the present economic conditions. Note that the ex–post *factor recovery* of an oil physical resource is a strongly variable fraction whose median is about 35%. This variability is very large for small reservoirs that are particularly frequent today. For this reason 35% of volumetric geological evaluations of oil resource might have been reasonably used in the past with large size reservoirs, but today does not seem appropriate. These uncertainties are increased by other factors. For instance, the United States Geological Survey, USGS (2000), proposes a very broad interpretation with the introduction of a revision principle based on "reserve growth". The variable weight choices of proven reserves are justified by different arguments;

c) *probable and possible reserves.* These refer to *undiscovered* petroleum and are based upon subjective assessments with different degrees of probability, usually very small (e.g. 5%).

Could we do the direct sum of these components to form the URR? Probably not. The risk of accounting duplications or omissions on pure virtual information may suggest more prudent approaches. A geologist would infer production and peak oil from physical evidence about the URR but the *estimated* URR is not a physical evidence.

In this paper we consider the URR simply as an unknown fraction of physical resource and infer a URR estimate which is derived from and is consistent with historical production under special diffusive models that incorporate modulations of economic and political factors.

The use of production data, combined with exogenous interventions, avoids the problem of an unreliable estimation of reserves and represents an innovative way to face this aspect.

The limited flexibility of "hubbertarian" models, on the one hand, and the problem of unreliable reserves evaluations on the other, have led to a possible solution firstly proposed by Guseo [20] and subsequently by Guseo and Dalla

Valle [1]. These works estimate the URR and the dates of oil depletion – at the peak – and at 90% of saturation using oil production cumulative data (oil stock data) with special controlling functions.

In this paper we extend the results obtained in [1] and [2] by referring to special Generalized Bass Models (GBM) first introduced by Bass *et al.* [21]. We include intervention variables useful for separating stochastic disturbances from systematic changes in life-cycle behaviour possibly due to price effects and strategic effects. Our purpose is to forecast world crude oil peak and global oil evolution of crude oil production in the following years, evaluating the relative significance of price shocks compared with technological or political upheavals.

Note that oil peak estimates are here exclusively global. Local applications of suitable versions of GBM were performed in [1] with reference to mainland U.S.A., Alaska (with Prudhoe Bay fields), Norway and Great Britain by demonstrating the flexibility and robustness of the proposed solution. In particular, GBM is not unimodal in nature. See, for instance, the Great Britain case in [1].

In Section 2, we briefly introduce the structure of the GBM, including some properties and some useful interpretations. Section 3 deals with some different choices about the intervention variables and compares the derived models with reference to the stochastic residual component. In Section 4 we test the performances of the models introduced in Section 2 and Section 3. Section 5 is devoted to discussion and final comments.

2 GBM diffusion model

A diffusion process may be described with different emphasis on moment components. In the stochastic differential approach the main effort is devoted to the second order aspect. In this article, the main interest is based upon the first order moment of a process because of its possible non stationarity due to initializing aspects, monotonicity and asymptotic character of the saturating effects of a life–cycle pattern.

The main ideas on this topic are based on models developed by Bass [22], [21], where the latter work extends the previous one by connecting the diffusion to external intervention factors. Both models may be considered as special cases of the Riccati equation [20], discovered by Count Jacopo Riccati (italian mathematician) in 1720, which, in turn, includes the logistic equation by Verhulst (1838).

The *diffusion of an innovation* in a social system may be described through the consumption choices of the agents. In particular, different behaviour in consumption reflects a different access to information. This idea was firstly proposed in a formal way by Bass who formulated a model with two classes of agents: the *innovators*, that give direct attention to advertising or communication of companies and the *imitators*, that adopt the innovation only in a subsequent time by reinforcing a personal opinion on the basis of a word–of–mouth effect. The interaction between these two classes makes possible a diffusion process that is more realistic than one represented by a simple logistic approach, explicitly considering an initializing process.

The Bass model describes the life-cycle of a generic product by a cumulative function $z = z(t)$. It is a function of time and potential market, m, (*carrying capacity*). On the basis of earlier stock data, it is possible to estimate the diffusion parameters – with different degrees of confidence – and in particular, the carrying capacity, m.

Let us denote by z' the instantaneous adoptions that are decomposed into two additive parts. The first one, $p(m - z)$, is referred to as *innovators* which adopt with a constant rate p over time. The difference $(m - z)$ depicts the residual market. The second component, $q(z/m)(m - z)$, is characterized by a delay in adoptions due to a word–of–mouth effect expressed by a simple ratio z/m that modulates the adoption rate q, which represents the accessibility of *imitators* to the residual market.

The carrying capacity, m, is not forced to be defined exogenously. It is simply a special unknown component of the diffusion process in a more general environment. For example, the potential market is only a subset of a susceptible population of consumers and, as we will see in Section 4, oil URR is only a subset of the corresponding Physical Resource.

In [21] the standard Bass model is extended with the introduction of a very general perturbation described by an integrable function $x(t)$. This function can vary around 1 and may represent political, economic and structural interventions. If $x(t) = 1$ we obtain the standard Bass model, BM.

The GBM is therefore

$$z' = m\left(p + q\frac{z}{m}\right)\left(1 - \frac{z}{m}\right)x(t) = \left(p + q\frac{z}{m}\right)(m - z)x(t), \tag{1}$$

and the general closed form solution is

$$z(t) = m\frac{1 - e^{-(p+q)\int_0^t x(\tau)d\tau}}{1 + \frac{q}{p}e^{-(p+q)\int_0^t x(\tau)d\tau}} = mF(t), \qquad 0 \leq t < +\infty. \tag{2}$$

Function $F(t)$ represents a modified Riccati distribution function (see [23]). Intervention function $x(t)$ allows a local modified perception of residual market, $(m - z)x(t)$. In other words, the function $x(t)$ modifies the geometry of time, and not the carrying capacity, m, or the intrinsic diffusion parameters p and q. This is very important in many fields. It can be easily proven (see Guseo [20]) that asymptotic quotas of innovators and imitators are not affected by $x(t)$. Modifications of $x(t)$ are effective only in the central part of a life–cycle.

3 Statistical modelling of GBM: hierarchy of components

Under the assumption that memory effects have a non–uniform distribution over time, we can model the intervention function $x(t)$ in Equation (1) through some exponential shocks, for example,

$$x(t) = 1 + c_1 e^{b_1(t-a_1)} I_{t \geq a_1} + c_2 e^{b_2(t-a_2)} I_{t \geq a_2} + c_3 e^{b_3(t-a_3)} I_{t \geq a_3}, \tag{3}$$

where $c_i,\ i = 1, 2, 3$, control depth and sign of perturbations, $b_i,\ i = 1, 2, 3$, describe effects persistency and $a_i,\ i = 1, 2, 3$, denote the starting times of exponential shocks. Note that usually parameters $b_i,\ i = 1, 2, 3$, are negative if memory is decaying to the stationary position (mean reverting), i.e., $x(t) = 1$. Sometimes they may be positive and this aspect introduces a strong acceleration in the saturation of a life–cycle.

Function $x(t)$ may be defined with special interest to implementation of exogenous variables, i.e., price variations or other possible impact variables in order to test their effects.

Local diffusions may also be anticipated or delayed randomly for a variety of reasons so that it is natural to include stochastic residual components in model building.

A simple specification of a statistical version of a GBM may be of nonlinear regressive nature, i.e.,

$$y(t) = f(\beta, t) + \varepsilon(t) = z(t) + \varepsilon(t), \tag{4}$$

where the deterministic part $f(\beta, t)$, equal to $z(t)$, is a nonlinear function of the unknown vector of parameters $\beta \in R^k$ and other time–dependent variables included in $x(t)$. The component $\varepsilon(t)$ is a stochastic process representing the i.i.d. residual error. The usual assumptions consider $\varepsilon(t)$ as a white noise process. A further specification is based on the assumption of normality.

Firstly, a simple nonlinear least squares parameters estimation method can be implemented directly (see for instance Seber and Wild [24]). In a second phase, we examine the residuals of nonlinear regression.

If residuals do not support the hypothesis of a white noise process it is possible to consider the well-known transfer function models (ARMAX), proposed by Box and Jenkins [25] in order to approximate dynamic relationships between an input x_t and an output z_t, see for instance [20].

Therefore, we estimate parameters β of Equation (4) following a non linear least squares procedure (e.g. Marquardt, Gauss–Newton or other criteria). At a second step, we use the estimated function $f(\hat{\beta}, t)$ instead of regressor x_t, or a lagged multiplicity of regressors, in an ARMAX model. The solution is clearly sub-optimal if compared with an estimation procedure which takes into account all the parameters jointly. The lack of fit is however quite limited.

In general, it is not a good choice to build a model in which a global optimizer determines simultaneously the most important components. Actually, this fact may generate a local good fitting with a confounding effect between deterministic and stochastic components.

In particular, in GBM modelling there is a prominent role of *natural diffusion* which provides a start–up of the process (innovative component) and iteratively initializes the imitative or logistic component which assures convenient curvature before saturation.

A second level in the model construction is necessarily devoted to the deterministic identification of systematic perturbation $x(t)$. This is not a minor problem. Such perturbations may strongly interact with the *natural diffusion* by shifting peaks, delaying or anticipating saturations.

Finally, some attention must be paid to the computational aspects of non-linear estimation with a considerable set of parameters. Initializing choices, interruption rules and convergency tests are not simple matters and require some joint evaluation of specific inference criteria.

4 World Oil Depletion Models: price effects and interventions

In the sequel we model crude oil production as a diffusion process directly controlled by correlated diffusion processes of technological innovations, such as transport, heating, electricity production, chemical applications, etc.. Here we emphasize the reasonable basic assumption that oil–consuming technologies follow documented diffusion processes with limited life–cycles which may compete and interact during their evolution. Spatial and temporal diffusions of such technologies are the basic and not unique drivers of oil production. The large scale oil extraction incorporates in a *learning process* new discoveries and *standard efforts* in technological improvements.

The carrying capacity, m, in this case represents the URR and is associated to the limiting behaviour of the cumulative production process. We have declared that the carrying capacity is one of the main aspects we are interested in because, apart from the physical existence of oil, the production process has to face up to all those technological, political and economic constraints that have qualitatively justified the distinction between "reserves" and "resources".

The regulatory or strategic interventions of political and economic nature that influence oil production – considered partially persistent and conceptually deterministic – may be modelled through the function $x(t)$. We underline that the proposed GBM modeling does not treat the issue of Oil Peak as "purely external" factor nor it is solely determined by the magnitude of total oil reserves in the ground. The GBM includes intervention function $x(t)$ which is modelled on an historical ground by external factors and its future behaviour is essentially free (see, for instance, final scenario simulations in Section 5).

The problem of data quality

Oil data are not of public domain and this imposes strong limitations on scientific evaluation and control. The mere definition of oil also seems to be time–dependent. If we look at BP (British Petroleum) data (2005) (see [26]), we observe that in the last two decades oil is not only crude oil but also shale oil, oil sands, NGL (*Natural Gas Liquids*) and others. This heterogeneous accounting process may give rise to instabilities in pattern recognition of production evolution.

Here, we examine daily world oil production in thousands of barrels, from 1900 to 2002. Data are provided by Industriedatenbank from 1900 to 1986 included. The second part of the series, 1987 - 2002, is based on BP (2003) data (see [27]) in order to recover the new contribution of NGL. The misleading or truly false information regarding over–accounting of crude oil reserves in financial transactions is fully ignored. The same source, BP, provides the series of barrel prices $D(t)$, with $t \in [1900-2002]$ in USA dollars (fixed base: 2002). This fact allows us to model at a first stage the intervention function $x(t)$ on the basis of price variations.

We have performed a wide number of transformations of price series $D(t)$ in order to explain systematic perturbations in the series $z'(t)$ of daily world oil production. Figure 1 may suggest many competing relationships.

We have considered various models for $x(t)$ following different economic reasonings. The main results are summarized in Table 1 where we report the models (D' is the function of prime difference of D), the R^2 and the residual sum of squares (SSE).

TABLE 1 ABOUT HERE

The performance is not completely satisfactory because an optimal configuration for this kind of models can be reached only when the R^2 is at least greater than 0.99995. A direct inspection of the graphical representation of both the observed data and each of the applied models gives strong evidence to reject them. Table 1 highlights the global poor fitting of case a). A good performance of cases b), c), d) and e) in the first part of the series from 1900 to about 1970 is compensated by a very poor fit in the sequel with an oscillating structure of prediction that is incoherent with data. Finally, cases f) and g) present a very high perturbation in the central part of the series.

FIGURE 2 ABOUT HERE

These results suggest that price series alone may not control completely expressed demand. Moreover, the direct examination of data, see Figure 1, suggests a self–evident contradictory behaviour: sharp positive spikes in prices correspond to local increment in expressed demand. This is incoherent with the standard economic theory of quantity–price relationship.

A second class of models is based on the hypothesis that the historical shocks (e.g. '73, '79) that emerged during the natural evolution of oil production must be included in the analysis in order to explain the major variations in oil production data. We do so by taking into account sequentially strategic shocks and then price effects.

The main results are summarized in Table 2 which reports the applied models for the intervention function $x(t)$, where $E_i = c_i \exp(b_i(t - a_i))$, the R^2 and the SSE. Sometimes, the regressive implementation of the model is improved with an ARMAX model based upon one regressor or more lagged regressors depending upon the predicted values of the first regressive step.

TABLE 2 ABOUT HERE

Here we skip some statistical improvements of interpretation based upon linearized confidence intervals even if some improvement could be obtained by considering simultaneous exact confidence regions (see, for instance, Guseo [28] and Bates and Watts [29]) in order to examine intrinsic curvature aspects of solution locus. Weighting criteria depending upon heteroscedastic dispersion proportional to instantaneous effects, $\sigma(t) = \sigma z'(t)$, have been used to improve goodness–of–fit performance with different local regimes.

By analyzing Table 2 we see that model a), discussed for the first time in [2], is statistically well fitted (note that the R^2 reaches a good level). In this framework the world oil peak is estimated to occur in 2006 and a 90% depletion time is located around 2041 under uniform and regular interventions in the period following 2002. If we compare these new results with those obtained in [1] we see that a further positive exponential shock emerged around 1964. In this sense the strong intervention after 1979 may be thought of as a compensating element in order to correct the system towards a standard regime. In particular, the coefficients c_i are similar but opposite in sign. The coefficients b_i are negative.

An instantaneous representation of observed and predicted values by models a), j) and k) is summarized in Figure 2.

FIGURE 3 ABOUT HERE

Models b) and c) are good improvements of residual autodependence structure. As we can see, the SSE decreases of a factor 10 but this type of modelling may interact with possible omitted systematic components of control function $x(t)$ as we will see later on. Model d) considers a simple combined effect of two shocks and the relative variation of prices $(ln(D))' = D'/D$. The partial contribution of price relative variation is not statistically significant. Models g) and h) highlight the effect of adding price series to the previous two shocks model by ratio $(1/D)'$ or both terms $(1/D)' + D'/D$. In these cases, the squared partial correlation is about $\tilde{R}^2 = 0.057$ ($F \simeq 5.267$) and, respectively, $\tilde{R}^2 = 0.116$ ($F \simeq 5.368$), denoting a weak significance of price effects after shock absorbtion of main perturbations. Improvements in models e) and f) follow the previous comments for similar situations. Model i) exhibits a good performance if compared with model g).

FIGURE 4 ABOUT HERE

GBM with 3 exponential shocks

The most interesting and surprising result is represented by model j) in which we directly attempt to accommodate three shocks. If we compare model a) with model j) we find that squared partial multiple correlation coefficient is $\tilde{R}^2 = 0.56$, ($F \simeq 17.06$), so that the evidence of a third shock is very strong. To see this compare Figure 3 with Figure 4. It is useful to summarize the main estimates in Table 3.

TABLE 3 ABOUT HERE

We note a structural change of the memory of interventions. All coefficients $b_i,\ i = 1, 2, 3$ are positive so that the effects are persistent in time and interact significantly with the normal evolution controlled by the standard Bass structure. The beginning times of the shocks are correctly positioned (1951, 1974-5, 1980-1) and this fact allows clear interpretations.

The starting point of a new global economic effort begins in 1951 after World War II and earlier crises such as that of 1929. Economic growth is sustained by an exponential positive deviation of crude oil production until 1974 (1973 for historical references). In 1973 we have the first long memory negative correction (Yom Kippur war and related embargo) followed by the second one in 1980 (1979, OPEC limitations and a decade of wars in the Middle East region). The consequences of this new modelling are impressive. The peak time of maximum instantaneous production, 2007, is shifted by one year with reference to the corresponding peak time based on a GBM with two exponential shocks described by model a). The effect of long memory perturbations is recognized in the identification of depletion times, $t_{0.90} = 2019$ and $t_{0.95} = 2023$, which appear particularly imminent.

If we look at these results from a statistical point of view, we deduce that in this case a Hubbert's approach based upon a *pure logistic equation* is too simplistic. Systematic perturbations have introduced a steady modification in the production framework.

In order to understand these effects, see Figure 2 where a heavy right tail plot belongs to a forecast based on a GBM with two exponential shocks with negative memory coefficients. The shorter forecasted curve is based upon a GBM with three exponential shocks and long memory effects sustained by positive coefficients.

In model k) we consider the marginal effect of prices added to a GBM with three shocks. The extension is very poor with a very low squared partial correlation, $\tilde{R}^2 = 0.01052$.

FIGURE 5 ABOUT HERE

The special position of the shock arising in 1951 is highlighted here, for the first time, through a GBM with three exponential shocks, extending Hubbert's pioneering work. See, for instance, Figure 5 where the dots represent the daily oil production per year, the broken line depicts the GBM model with three shocks and the continuous line represents the Hubbert–Bass process without interventions.

The subtle and shifty deviation originated in the post–war period beyond 1951 has generated an incremental consumption of oil not fully balanced by the feedbacks of 1970's. This deviation from the Hubbert–Bass model is recovered by the three shocks GBM with an abrupt contraction of the diffusion process that is quite severe. The present model forecasts a URR of 1524 Gbo (Giga barrels of oil), a peak positioned in 2007 with a production of 76.34 mb/d. A depletion of 90% URR in 2019 with a production of 55.33 mb/d and a 95% saturation during 2023 with a production of 36.13 mb/d are estimated under a uniform scenario for intervention function beyond 2002, ($x(t) = 1$).

On one hand, the severity of contraction depends upon the assumptions of proposed model and the specific data set. The stability and quality of data has been mentioned above. We have noted that such sources tend to modify information from year to year. Compare for instance BP (2003) with BP(2005) data, [27], [26]: the introduced changes may generate instabilities of different nature.

On the other hand, this URR estimate is surely due to an important characterizing aspect of the model. The estimated three shocks satisfy a balance equation. The anomalous long deviation in production, observed and significantly identified for more than two decades starting from 1951, has to be considered as an effective component of URR. Anticipated extraction is only subtracted from the future.

5 Final remarks, scenario evaluations and discussion

The explicative or anticipative role of annual average price of crude oil has a weak statistical significance. Both models h) and k) present a negative sign in price coefficients that is coherent with the standard negative relation quantity–price. Nevertheless, the analysis based on model j), i.e., GBM with three shocks, excludes a central role of prices of crude oil in determining the decisions of its use. Crude oil is the primary energy source of this short historic period (one century and half) and its life time is limited.

FIGURE 6 ABOUT HERE

The structural change of the sign (positive) of coefficients $b_i,\ i = 1, 2, 3$, highlights the central role of strategic interventions with respect to the normal evolution of the production process. The acceleration of the "learning" dynamics of the social and economic system is a stable modification of the standard Bass trend. This modification is permanent and not absorbable as in a usual decay process, where the memory of an event has negative coefficients.

The actual oil production, after two negative shocks, is still under the influence of the first shock (1951) which started with the first post–war period.

This push, added to the natural diffusion, gives rise to a contraction of the right tail of distribution over time compared with GBM estimated models with one or two exponential shocks. The peak time is delayed, 2007, but saturation is strongly anticipated, under a uniform assumption on $x(t)$ beyond 2002, $t_{0.90} = 2019$ and $t_{0.95} = 2023$.

It may be useful to evaluate the sensitivity of the proposed forecasting procedure by modifying the assumption of uniformity of function $x(t)$.

FIGURE 7 ABOUT HERE

As a first hypothetical scenario we may expect that after the oil peak (2007) some international political action may be positively introduced. If we assume two interventions, similar to 1973 and 1979 shocks, located around 2008 and 2013 we observe that oil production is shifted to the right by three or four years (see Figure 6). This strategy may be useful to improve current technological alternative solutions for energy in a more suitable way. This requires widespread consensus on various limitations at social and economic levels.

By contrast, a second hypothetical scenario may be based on a simpler philosophy, i.e., an inertial trend "business as usual" similar to the 1951 long memory positive shock. Under this hypothesis we have a right-shifted peak of one year (2008) followed by a steeper descent production with a contraction of one or two years of depletion times $t_{0.90}$ or $t_{0.95}$ (see Figure 7). Substitutions and migrations to alternative energy sources seem to be more cumbersome.

A partial motivation of previous scenario hypothesis may be based upon increasing energy consumption within Asia Pacific area and, in particular, in

China. In order to appreciate this effect see Figure 8 with reference to oil.

FIGURE 8 ABOUT HERE

Our results may be usefully compared with some contributions in literature.

USGS (2000), [30], presents three possible scenarios (95%, mean, 5%) based upon different assumptions regarding the URR definition and certainty levels of sources and its configurations. USGS scenarios are not based on a historical learning of dynamics in extraction (production) process but on partially subjective averaging of the URR components with possible duplications. The order of such "assessments" are respectively $2.25, 3.0$ and 3.9 Gbo. Oil definition in this case includes high density oil, deep–water up to a depth of 4000 metres.

BP (2005) reports 1188 Gbo oil reserves at the end of 2003 so that the URR may be estimated to be about 2200 Gbo.

Bakhtiari in [33] describes the main results of the WOCAP model with a world oil peak estimated around 2006–07 and an associated production likely by 81 mb/d. The model, developed upon oil reserve estimates by Colin J. Campbell, predicts global production of conventional oil for all the hydrocarbon liquids, such as NGL, etc.. Estimation of daily production of 2020 is 55 mb/d. Note that our results are essentially equivalent.

In [32] the issue of oscillation of oil prices during the shocks occurred in 1973 and 1980 is addressed by Bakhtiari. The main result is an unpredictable relationship between demand and supply. Support is given to a common effect of those shocks, i.e., a transition to low–energy devices (computers, electronics, robotics, bio–engineering, etc.) as compared with traditional industrial products, steel–car, chemicals, etc.. The role of the "well–informed spectators" of the oil game, i.e., journals, specialized consultants, institutes, is important as they "massage the message" and, clearly, influence oil price–settings. This result contributes to enhance our work which highlights the surprisingly weak relationship between observed prices, oil demand and supply. The decade post 1986 is relatively stable with some exceptions that confirm paradoxical behaviours of the quantity–price relationship.

In [34] the significance of a structural change in the Pindyck model is tested by Bernard *et al.*. Dynamics of oil prices are based upon an extension of an AR(1) process (a structural ARMAX). The "nested to the boundaries" fixed coefficients sub–model is proved to be sufficient even if no deeper analysis is performed. A raw control reveals that the reduced model gives rise to a low level for R^2, namely $R^2 = 0.81$. It is perhaps surprising that this use of very complex models does not lead to much gain in significance.

Bentley examines in [31] a conventional oil modelling by Campbell and Laherrère and the corresponding checking provided by the University of Reading. The Hubbert model is the basis but with known flaws. In particular, dynamics of Alaska production may be a counterexample, that is not examined. The *technological shock* due to the accelerated extraction of oil from the Prudhoe Bay may be recognized with a more flexible tool and not with a simple logistic model. Very curious is the peak shifting of 10 years to compensate the reduction of demand following the 1970's oil shocks. However, there is no mention of a positive shock since 1951 that have strongly modified the basic dynamics of oil production by postponing the peak and, obviously, by contracting depletion times.

The nature of forecasting errors in this area is discussed by Lynch in [35] and [36] with reference to geophysical models and, in particular, to the Hubbert's model. Obviously, pure Hubbert's model has no room for oil economics since the basic differential equation is the classical logistic. The under–estimation of the Hubbert's model is a known aspect but no analysis is given in order to introduce more flexible tools able, for instance, to track an intervention or a systematic perturbation. Lynch supposes that the origin of the problem relies on the strong assumption of a static URR. The impact of technology in expanding resources is sometimes reasonable but rare systematic interventions may affect normal dynamics. In order to make an example, the forced production of Prudhoe Bay in Alaska was a positive shock, while, on the contrary, the negative effect of accidents in pipelines (Piper Alpha disaster) in the North Sea depressed regular production. These accidents led to a restructuring of UK offshore safety legislation followed by major changes to the Petroleum Reven Tax. These radical changes are not recognized by a simple Hubbert model.

A solution to these claims is firstly discussed in [20] and [1]. This work emphasizes the prominent role of GBM in the analysis of world crude oil time series production with persistent effects of rare interventions on this special life–cycle. The predictive role of crude oil annual average price is weakly statistically significant demonstrating that oil price elasticity is quite limited during the XXth century (under deflated dollars, (2002)).

Who is driving the trend of oil production? The answer is surely complex but we think that diffusion of knowledge and corresponding oil consuming technologies in the social system play a key role. Adoption decisions may be interpreted and partially governed but effective choices are the result of local cultures and lifestyles, that evolve and change in time. As a matter of fact, the ratio q/p drives the limiting temporal behaviour of the fraction Q of imitative decisions in a GBM (see [20]). Within three shocks GBM (see Table 3), the ratio q/p is about 608 and the corresponding asymptotic imitative fraction Q depicts a dominant effect, $Q > 99\%$. Emerging global behaviour is just the systematic representation of these evolutive patterns, whose complexity may be understood through the interaction – not the separation – of different competencies.

Acknowledgements

We are grateful to Dr. Colin J. Campbell, Dr. A.M.S. Bakhtiari, Prof. U. Bardi, Prof. Douglas B. Reynolds and Prof. Stuart Coles for their suggestions and comments about a preliminary version of this work. We thank also anonymous referees giving precise comments and suggestions in order to highlight better the world oil depletion problem within the dynamics of energy systems.

References

[1] R. Guseo, A. Dalla Valle, Oil and Gas Depletion: Diffusion Models and Forecasting under Strategic Intervention, Statistical Methods and Applications 14, (2005) 375–387.

[2] M. Guidolin, Cicli energetici e diffusione delle innovazioni. Il ruolo dei modelli di Marchetti e di Bass, (Thesis), University of Padova, Italy, 2004.

[3] J.E. Cohen, Human Population: The Next Half Century, Science 203, (2003) 1172–1175.

[4] C. Marchetti, On 10^{12}: A Check on Earth Carrying Capacity for Man, RR–

78–7, International Institute for Applied System Analysis, Laxenburg, Austria, 1978.

[5] C. Marchetti, Society as a learning system: discovery, invention and innovation cycles revisited, Technological forecasting and social change 18, (1980) 267–282.

[6] C. Marchetti, Logos, il creatore di imperi, Systema Naturae 5, (2003) 307–321.

[7] G.T.T. Molitor, From my perspective. Five economic activities likely to dominate the new millennium V. New atomic age, Technological forecasting and social change 70, (2003) 473–487.

[8] C. Hall, P. Tharakan, J. Hallock, C. Cleveland, M. Jefferson, Hydrocarbons and the evolution of human culture, Nature 426, (2003) 318–322.

[9] E. Morse, A. Jaffe, Strategic energy policy challanges for the 21st century, Council on Foreign Relations Press, New York, 2001.

[10] C.J. Campbell, A Guide to Determining the World's Endowment and Depletion of Oil, (http://www.hubbertpeak.com).

[11] M.K. Hubbert, Energy from fossil fuels, Science 4, (1949) 103–109.

[12] M.K. Hubbert, Energy Resources. A Report to the Committee on Natural Resources: National Accademy of Sciences, National Research Council Publication, 1000-D, Washington DC 54, (1962) 61–67.

[13] P.F. Verhulst, Notice sur la loi qui la population suit dans son accroissement, Correspondence Mathématique et Physique 10, (1838) 113–121.

[14] C.J. Campbell, J.H. Laherrère, The End of Cheap Oil, Scientific American, March, (1998) 78–83.

[15] D.B. Reynolds, Mineral Economy: How Prices and Costs Can Falsely Signal Decreasing Scarcity, Ecological Economics 31, (1999) 155–166.

[16] D.B. Reynolds, Using non–time–series to determine supply elasticity: how far do prices change the Hubbert curve? OPEC Review, (2002) 147–167.

[17] J.H. Laherrère, Modelling future liquids production from the extrapolation of the past and from ultimates, ASPO, Uppsala, May 23 2002, 2002.

[18] J.H. Laherrère, Modelling future oil production, population and the economy, ASPO, Paris, may 26–27 2003, 2003.

[19] U. Bardi, The mineral economy: a model for the shape of oil production curves, Energy Policy 33, (2005) 53–61.

[20] R. Guseo, Interventi strategici e aspetti competitivi nel ciclo di vita di innovazioni, Working Paper Series, 11, Department of Statistical Sciences, University of Padova, 2004.

[21] F.M. Bass, T. Krishnan, D. Jain, Why the Bass model fits without decision variables, Marketing Science 13, (1994) 203–223.

[22] F.M. Bass, A new product growth model for consumer durables, Management Science 15, (1969) 215–227.

[23] R. Guseo, Organizzazione statistica dell'informazione e scelte di gestione: Teoria, Tecniche e Mini–Stage Aziendali, 2nd edition, CEDAM, Padova, 2004.

[24] G. Seber, C. Wild, Nonlinear Regression, Wiley, New Jork, 1989.

[25] G.E.P. Box, G.M. Jenkins, Time series analysis: forecasting and control, Prentice Hall, New Jersey, 1976.

[26] British Petroleum, BP Statistical Review of World Energy;(www.bp.com), London, 2005.

[27] British Petroleum, BP Statistical Review of World Energy;(www.bp.com), London, 2003.

[28] R. Guseo, Confidence regions in non–linear regression, in Proceedings of The 44-th Session of ISI, Madrid 1, (1983) 333–336.

[29] D. Bates, D. Watts, Nonlinear Regression Analysis and its Applications, Wiley, New Jork, 1988.

[30] United States Geological Survey, USGS, World Petroleum Assessment 2000, (www.usgs.gov).

[31] R.W. Bentley, Global oil and gas depletion: an overview, Energy Policy 30, (2002) 189–205.

[32] A.M.S. Bakhtiari, The price of crude oil, OPEC Review, (1999) 3–21.

[33] A.M.S. Bakhtiari, World oil production capacity model suggests output peak by 2006–07, Oil and Gas Journal, Apr. 26, (2004) 18–20.

[34] J.T. Bernard, L. Khalaf, M. Kichian, Structural Change and Forecasting Long–Run Energy Prices, Bank of Canada Working Paper 2004-2005, 2004.

[35] M.C. Lynch, Forecasting oil supply: theory and practice, The Quarterly Review of Economics and Finance 42, (2002) 373–389.

[36] M.C. Lynch, The New Pessimism about Petroleum Resources: Debunking the Hubbert Model (and Hubbert Modelers), Minerals and energy 18, (2003) 21–32.

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Table 1

World oil depletion: GBM models with pure prices control.

n.	$x(t)$ control model	R^2	SSE
a):	$1 + cD - a$	0.999624	2.25264E10
b):	$1 + aD'/D$	0.999822	3.84188E09
c):	$1 + aD'$	0.999813	1.11859E10
d):	$1 + cD - a + bD'/D$	0.999892	6.49115E09
e):	$1 + aD' + bD'/D$	0.999820	1.07567E10
f):	aD	0.978043	1.31507E12
g):	a/D	0.988635	6.80707E11

Table 2

World oil depletion: GBM models with shocks and prices control.

n.	name	$x(t)$ control model	R^2	SSE
a):	gbm2e	$1 + E_1 + E_2$	0.999988	7.24271E08
b):	gbm2e, armax(4,4)	$1 + E_1 + E_2$, lag = 0		6.25567E07
c):	gbm2e, armax(2,0)	$1 + E_1 + E_2$, lag = 2		6.79209E07
d):	gbm2elnD	$1 + E_1 + E_2 + a(ln(D))'$	0.999988	7.06147E08
e):	gbm2elnD, armax(4,4)	$1 + E_1 + E_2 + a(ln(D))'$		7.48614E07
f):	gbm2e+f(D), armax(4,0)	$(1 + E_1 + E_2)' + D' + (ln(D))'$ inst. data		6.77601E07
g):	gbm2eDra	$1 + E_1 + E_2 + a(1/D)'$	0.999989	6.83253E08
h):	gbm2eDralnD	$1 + E_1 + E_2 + a(1/D)' + b(ln(D))'$	0.999989	6.39745E08
i):	gbm2eDrarun	$1 + E_1 + E_2 + a(1/D)$	0.999989	6.62275E08
j):	gbm3e	$1 + E_1 + E_2 + E_3$	0.999994708	3.16947E08
k):	gbm3+1/D+lnD	$1 + E_1 + E_2 + E_3 + a(1/D)' + b(ln(D))'$	0.999994764	3.13612E08

Table 3

World oil depletion: GBM estimates with three exponential shocks.

m = 4174561	p = 0.00010439	q = 0.063497
c_1 = -0.3021860	b_1 = 0.05674	a_1 = 80.50
c_2 = 0.0717753	b_2 = 0.07187	a_2 = 51.07
c_3 = -0.2272032	b_3 = 0.07098	a_3 = 74.60

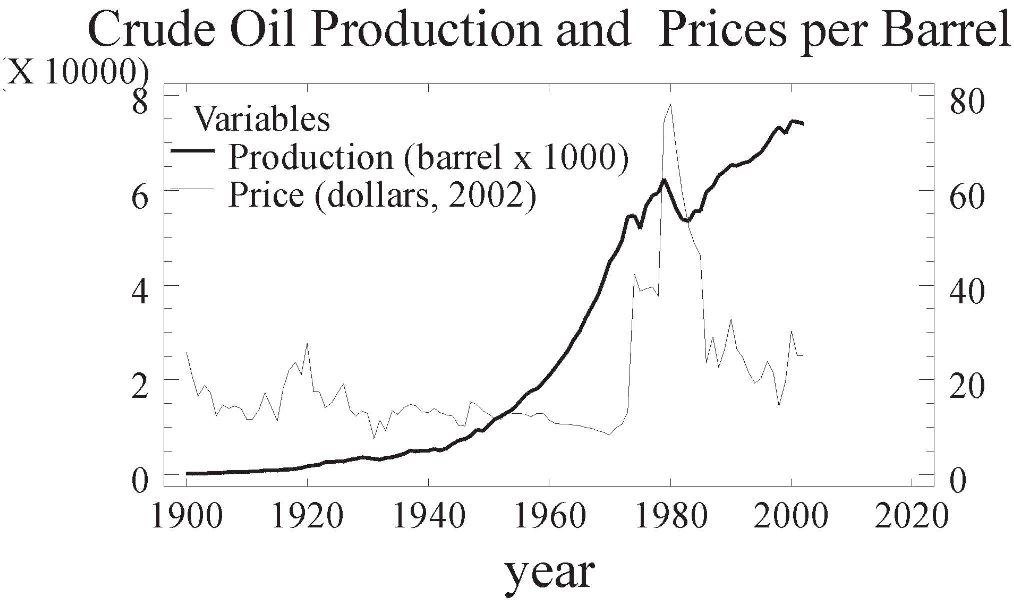

Fig. 1. World oil daily production (thousands of barrels) and prices per barrel (dollars, 2002); Source: Industriedatenbank and British Petroleum, BP.

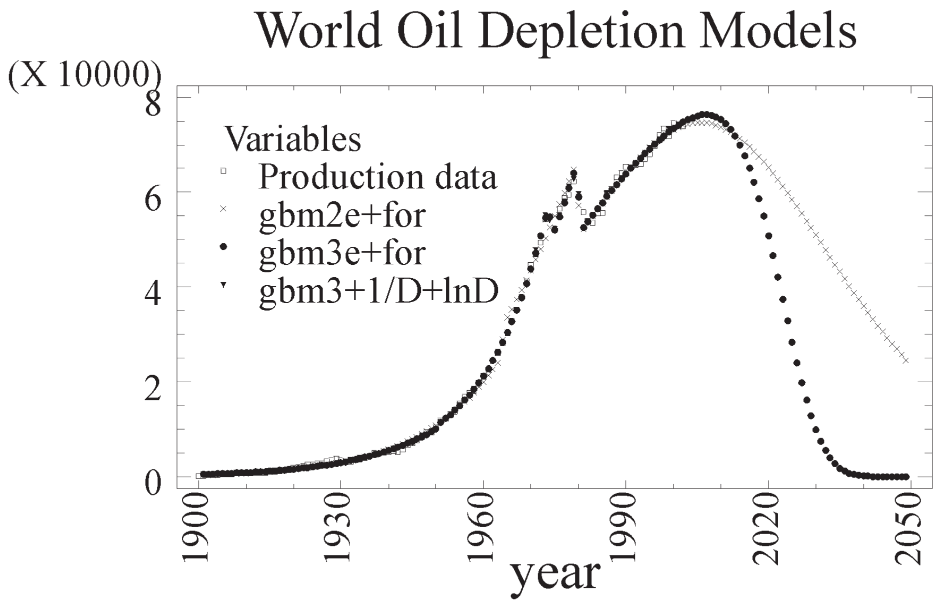

Fig. 2. World Oil Depletion Models: GBM under two or three shocks and price effects.

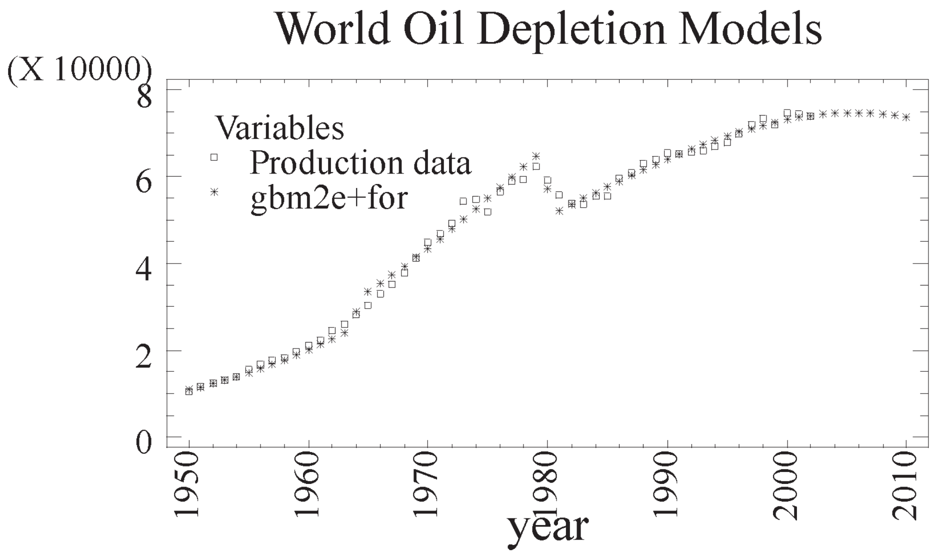

Fig. 3. World Oil Depletion: a zoom on GBM after 1950 with two exponential shocks.

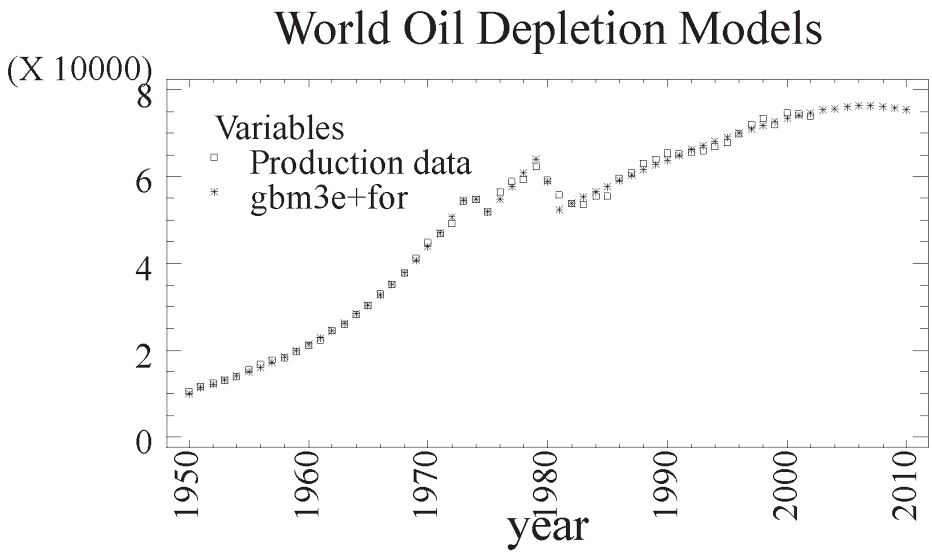

Fig. 4. World Oil Depletion: a zoom on GBM after 1950 with three exponential shocks.

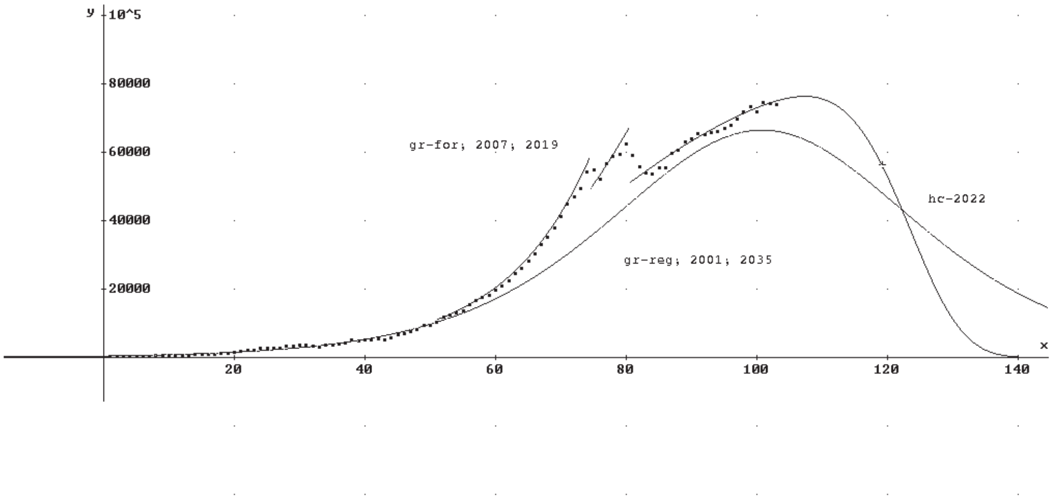

Fig. 5. World Oil Depletion: GBM with three shocks vs Hubbert–Bass model.

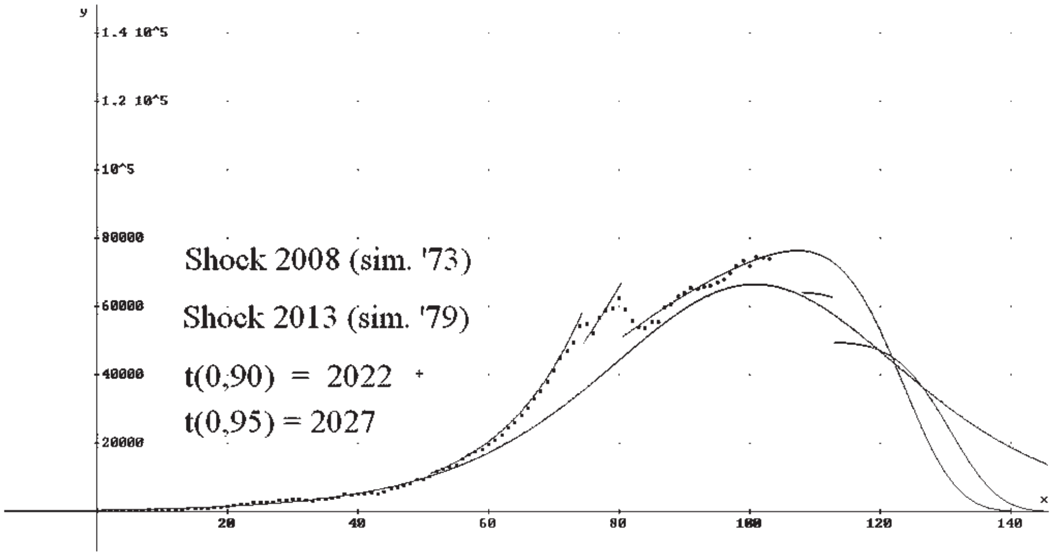

Fig. 6. World Oil Depletion: GBM with three shocks vs Hubbert–Bass and vs two capacity limitation constraints scenario similar to 1973 and 1979 shocks.

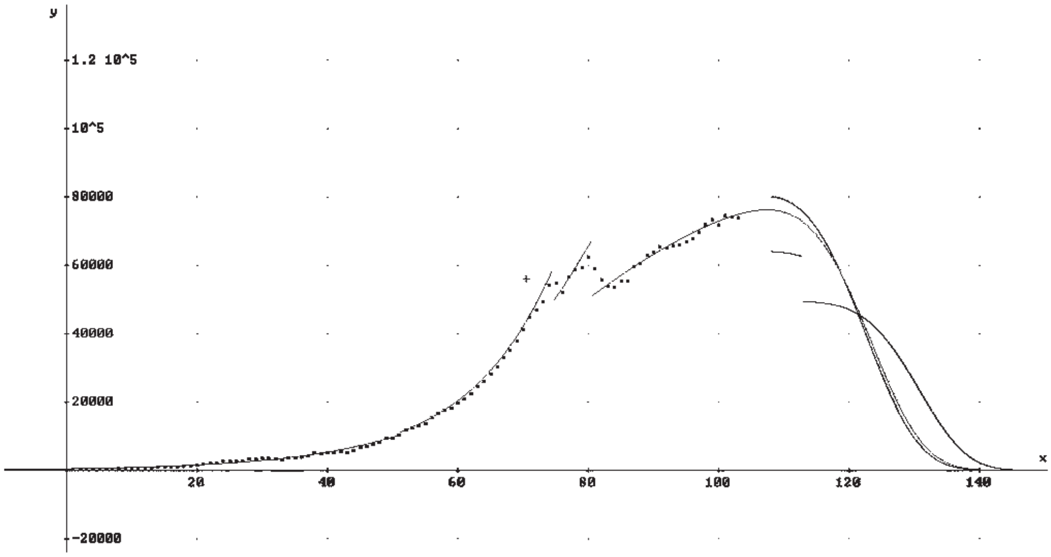

Fig. 7. World Oil Depletion: GBM with three shocks vs an increasing smooth expansion with no limitation and similar to 1951 positive shock.

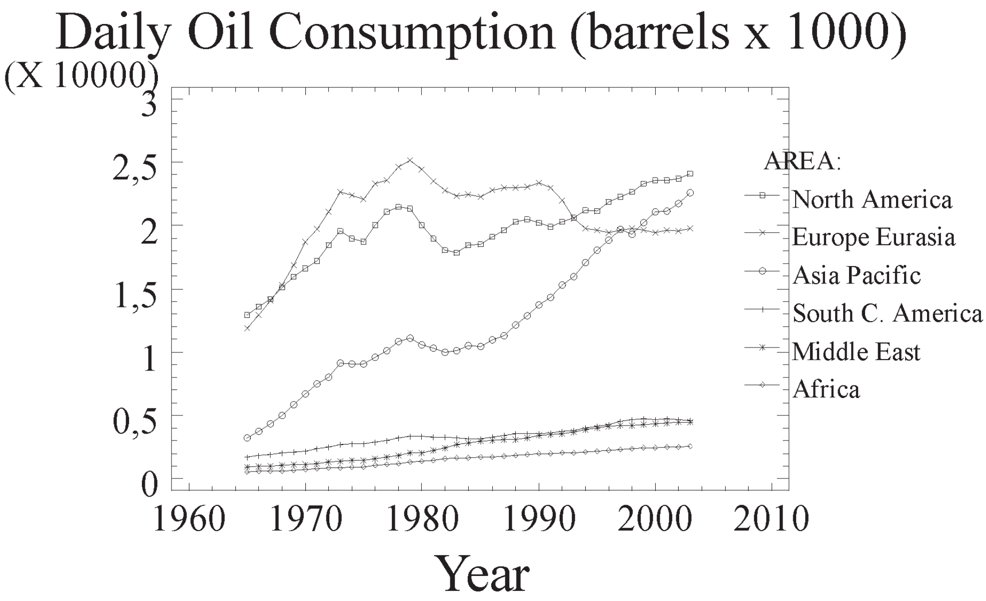

Fig. 8. Area oil consumption (thousand barrels daily); Source: BP (2003).